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Characterization of rate-dependent loading/unloading dynamic constitutive behavior of aluminum alloy by Lagrange experiment

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Demands for characterization of dynamic constitutive behavior of materials in high strain rate have grown in both scientific and technological standpoint. Traditionally, the construction of the constitutive equation should be based upon some empirical or phenomenological constitutive forms. This method is not only expensive in determining experimentally the model parameters, but offers no guarantee of uniqueness. An alternative innovative approach to establishing dynamic constitutive equation is called as Lagrange analysis method which was introduced and developed by Fowles and Williams [1], Cowperthwaite and Williams [2], Grady [3], and Seaman [4]. Upon the records from a series of stress or velocity gages in test materials through which one-dimensional plane waves are passing, the constitutive relationships among stress, strain, and strain rate can be calculated. In this method, no assumptions are made about the form of the constitutive equation, the steady state, and the extent to which the stress is an equilibrium stress. Thus, the method is completely general. In this letter, the Lagrange analysis method was used to characterize the rate-dependent dynamic constitutive behavior of LY12 aluminum alloy, and a set of usable loading/unloading constitutive equations was presented.

The symmetrical planar plate impact tests were performed by means of a 101-mm bore single stage gas gun and the stress history was monitored by manganin piezoresistance gages embedded in different positions in the specimen. The experimental set up is shown in Fig. 1. The impact velocity was in the range of 100–500 m/s. The impact of the flyer plate generates compression wave loading the test material, and this wave is reflected at the rear interface of the specimen into rarefaction wave unloading the material. After processing the data by the Lagrange analysis method [4, 5], the constitutive equations can be obtained. The curves of the stress-strain and the strain rate-strain are shown in Fig. 2. It can be seen from Fig. 2 that when stress reaches the peak stage, the strain rate does not return to zero, and a small stress increment during this stage induces a significant plastic flow.

The calculated stress-strain trajectories for the four tests are given in Fig. 3. It is shown from Fig. 3 that all the stress-strain curve show obvious hysteresis and a high stress level corresponds to a higher strain rate. There is a linear relationship between the peak stress and the logarithm of the peakstrain rate, as shown in Fig. 4. The slope of the line is defined as the strain rate sensitivity, which has the value of 1.714. From these results, it can be found that the constitutive behavior of the present test materials LY12 aluminum alloy is strain rate dependent.

As shown in Fig. 3, the transient stress is related not only to the transient strain and transient strain rate, but also to histories of the strain and strain rate. Since there is no simple algebraic equations among σ, ε, and $\dot{\varepsilon}$, the rate type quantity R is introduced for characterizing the rate-dependent constitutive behavior, which is given by

$$R = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon}\right|_{\mathrm{h}} = \frac{\dot{\sigma}}{\dot{\varepsilon}} \tag{1}$$

where $R(\varepsilon, \dot{\varepsilon})$ is the gradient of stress-strain curve and the ratio of the stress rate to the strain rate. It identifies the rigidity of materials.

Since there is a difference between loading behavior and unloading behavior, the modulus function $R(\varepsilon, \dot{\varepsilon})$ can be given as follows:

$$\text{loading: } R = R_0\left[1 - A(\varepsilon)\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}^*}\right)^{-\mathrm{m}}\right] \tag{2}$$

$$\text{unloading: } R = R_{\mathrm{u0}}\left[1 + R_{\mathrm{u1}}\varepsilon - B(\varepsilon)\left(-\frac{\dot{\varepsilon}}{\dot{\varepsilon}^*}\right)^{-\mathrm{n}}\right] \tag{3}$$

where $\dot{\varepsilon}^* = 10^3\ \mathrm{s}^{-1}$. $A(\varepsilon)$ and $B(\varepsilon)$ are the fitted functions and can be determined by the experimental data. From regression of the data of R, $\dot{\varepsilon}$, the parameters in Equations 2 and 3 can be obtained:

$$R_0 = 73.7\ \mathrm{Gpa} \quad m = 1.04 \quad R_{\mathrm{u0}} = 55.9\ \mathrm{Gpa}$$

$$R_{\mathrm{u1}} = 24.4 \quad n = 5.53$$

$$A(\varepsilon) = 1.42 - 314\,\varepsilon - 2.28 \times 10^3\,\varepsilon^2$$

$$B(\varepsilon) = -1.29 \times 10^3 + 4.75 \times 10^4\varepsilon + 628\varepsilon^2$$

According to Equation 1, the incremental form of rate-type dynamic constitutive equation can be

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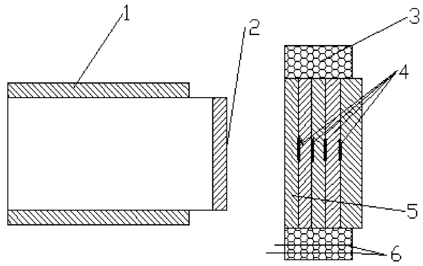

Figure 1 Schematic diagram of plate impact test: 1. barrel; 2. flyer; 3. epoxy; 4. stress gage; 5. target specimen; 6. pins.

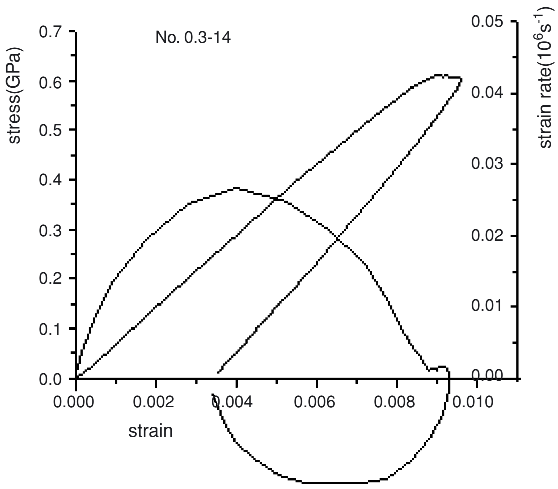

Figure 2 Relationship among σ, $\dot{\varepsilon}$ vs. ε.

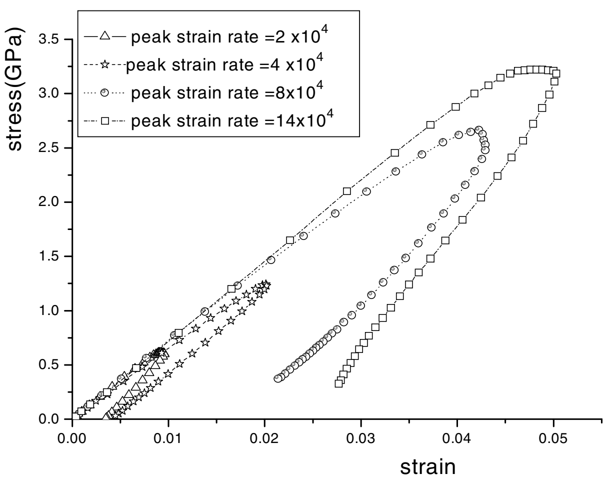

Figure 3 Stress-strain curves for four loading levels.

written as

$$\dot{\sigma} = R(\varepsilon, \dot{\varepsilon})\dot{\varepsilon} \quad (4)$$

When $t = 0$, $\sigma = 0$, $\varepsilon = 0$, the flow stress in the material can be given by

$$\sigma = R_0\varepsilon + R_0\int_0^t A(\varepsilon)\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}^*}\right)\dot{\varepsilon}\,\mathrm{d}t, \quad \mathrm{d}\varepsilon > 0 \quad (5)$$

$$\sigma = \sigma|_{\varepsilon=\varepsilon_\mathrm{m}} + R_{\mathrm{u}0}\left[(\varepsilon - \varepsilon_\mathrm{m}) + R_{\mathrm{u}1}\left(\varepsilon^2 - \varepsilon_\mathrm{m}^2\right) - \int_{t_\mathrm{m}}^t B(\varepsilon)\left(-\frac{\dot{\varepsilon}}{\dot{\varepsilon}^*}\right)^{-\mathrm{n}}\mathrm{d}t\right], \quad \mathrm{d}\varepsilon < 0 \quad (6)$$

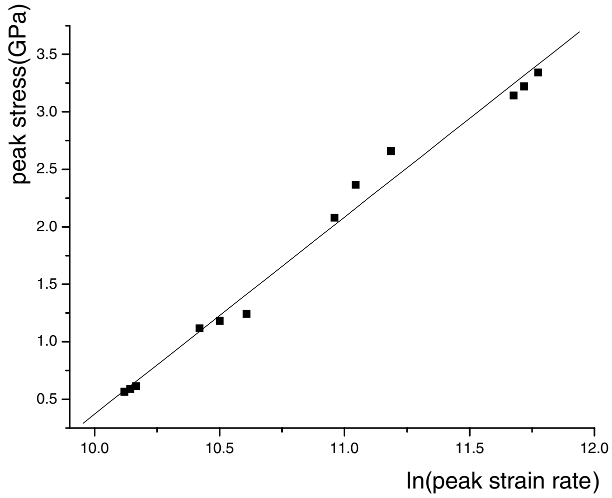

Figure 4 Relationship between the peak stress and logarithm of the peak strain rate.

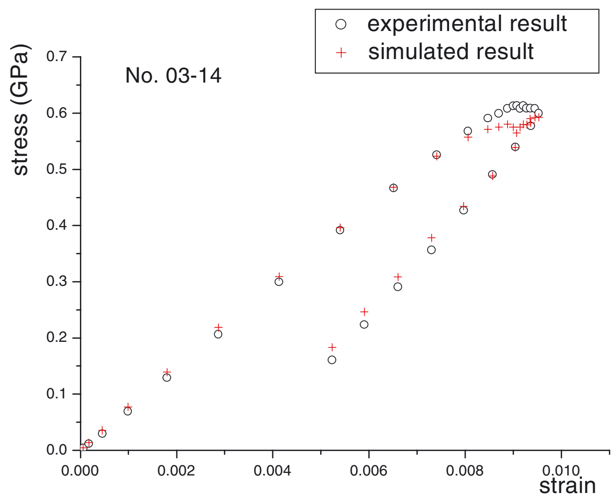

Figure 5 σ-ε curve comparison of simulated value with the experimental value.

where ε_m, t_m are the values at $\dot{\varepsilon} = 0$, respectively. The first term in Equation 5 is a linear response. The second term indicates the combined result of the softening behavior from large plastic deformation and hardening behavior from high strain rate. R_0 is a saturation value of the modulus function under high strain rate loading.

To demonstrate the utility of the proposed loading/unloading dynamic constitutive equations, a comparison of the calculated result according to the Equations 5 and 6 with the experimental result was made, which is shown in Fig. 5. It can be seen from Fig. 5 that the simulated result is in good accordance with the experimental result and the average relative error is less than 8%, 11% during loading and unloading.

In summary, the loading/unloading dynamic behavior of Ly12 aluminum alloy was investigated by Lagrange experiment. According to the measured stress records and Lagrange analysis method, a set of usable loading/unloading rate-dependent dynamic constitutive equations was constructed in this paper.

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